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Design of H-infinity Controller for A Linear Spring Connected Double Inverted Pendulum

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Abstract

A modified double inverted pendulum – modified by connecting the mass carrying the pendulum with another mass through a spring - makes the general inverted pendulum become a more interesting problem. The system is defined as a linear spring connected double inverted pendulum as proposed by Hou et al. [1],[2]. The system is highly nonlinear and unstable. However, the system can be simplified to a linear control problem through the linearization of a pre-specified equilibrium point such as the upright position of the double pendulum. Therefore, the linearized system allows the designer to apply various techniques of control methods to stabilize the system such as classical PID controller, LQR, etc. Practically, the system is unavoidably affected by an exogenous disturbance. The robust control technique is an appropriate method to deal with this situation. Also, if the energy of the disturbance is bounded, therefore, one can apply the popular robust H infinity control to solve this problem. Then, the H infinity controller is applied to an example of the linear spring connected double pendulum compared with LQR method through simulation.

Keywords: H infinity control; optimal control a linear spring connected double inverted pendulum.

1. Introduction

A modified double inverted pendulum – modified by connecting the mass carrying the pendulum with another mass through a spring - makes the general inverted pendulum become a more challenging problem. The system is unstable and highly nonlinear as general double inverted pendulum systems [1], [2] and [7]. Therefore, the system can be used as a test problem for various control techniques. Even though the system is nonlinear, a linearization can be used to simplify the system as a linear time invariant system at an equilibrium point; upright position of the double inverted pendulum. This approximation allows many linear control techniques to be applied to control and

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stabilize this system. However, in practicality, the control system will suffer and will be affected by the disturbance. In a situation where the energy of disturbance is bounded, the appropriate controller to stabilize a linear time invariant system under disturbance is H infinity controller. This method was used by Tsachouridis et al.[4] to stabilize the triple inverted pendulum system.

2. A Linear Spring Connected Double Inverted Pendulum System

2.1 Mathematical Model

A linear spring connected double inverted pendulum as purposed by Hou et al.[1, 2] and Hongxing [5] as shown in figure1. The system contains 4 degree of freedoms which are angles of double inverted pendulum and displacements of carts. The mathematical model representing behavior of the system can be constructed based by using Lagrange equation or Newton's Law as shown in Eq. (1) ,Hou et al.[1, 2].

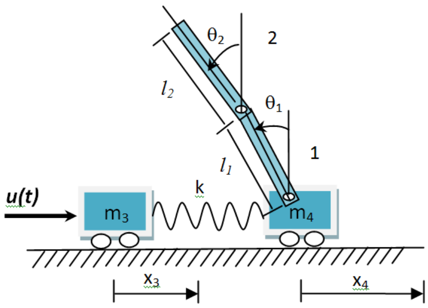


Fig. 1. A Linear spring Connected Double Inverted Pendulum System

$$-m_2 l_{C2}\cos\theta_2\ddot{x}_4 + m_2 l_1 l_{C2}\cos(\theta_1-\theta_2)\ddot{\theta}_1 + (J_{C2}+m_2 l_{C2}^2)\ddot{\theta}_2$$
$$= m_2 g l_{C2}\sin\theta_2 + \sin\theta_2 + m_2 l_1 l_{C2}\sin(\theta_1-\theta_2)\dot{\theta}_1^2 - c_2(\dot{\theta}_1-\dot{\theta}_2)$$

$$-(m_1 l_{C1}+m_2 l_1)\cos\theta_1\ddot{x}_4 + (J_{C1}+m_1 l_{C1}^2+m_2 {l_1}^2)\ddot{\theta}_1 + m_2 l_1 l_{C2}\cos(\theta_1-\theta_2)\ddot{\theta}_2$$
$$= m_1 g l_{C1}\sin\theta_1 + m_2 g l_1\sin\theta_1 - m_2 l_1 l_{C2}\sin(\theta_1-\theta_2)\dot{\theta}_2^2 - c_1\dot{\theta}_1 - c_2(\dot{\theta}_1-\dot{\theta}_2)$$

$$(m_4+m_1+m_2)\ddot{x}_4 - (m_1 l_{C1}+m_2 l_1)(\mu\sin\theta_1+\cos\theta_1)\ddot{\theta}_1 - (\mu m_2 l_{C2}\sin\theta_2 + m_2 l_{C2}\cos\theta_2)\ddot{\theta}_2$$
$$= \mu(m_1 l_{C1}+m_2 l_1)\dot{\theta}_1^2\cos\theta_1 + \mu m_2 l_{C2}\dot{\theta}_2^2\cos\theta_2 - (m_1 l_{C1}+m_2 l_1)\dot{\theta}_1^2\sin\theta_1 - m_2 l_{C2}\dot{\theta}_2^2\sin\theta_2$$
$$-k(x_4-x_3) - \mu(m_4+m_1+m_2)g - c\dot{x}_4$$

$$m_3\ddot{x}_3 = F(t)+k(x_4-x_3)-\mu m_3 g - c\dot{x}_3 \tag{1}$$

where m_3 = mass of cart 3, m_4 = mass of cart 4, c_0 = friction factor, c = tack coefficient, and $u(t)$ = control input.

2.2 Problem Statement

By letting $z_1 = x_3$, $z_2 = x_4$, $z_3 = \theta_1$, $z_4 = \theta_2$, $z_5 = \dot{x}_3$, $z_6 = \dot{x}_4$, $z_7 = \dot{\theta}_1$,and $z_8 = \dot{\theta}_2$,the equation of motion in Eq. (1) can be converted in the form of a first order differential equation, $\underline{z} = f(\underline{z}(t), u(t))$ where

$\underline{z}(t) = [z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8]^T$. Approximating by linearization, the linear system can be expressed in the state space form as

$$\dot{\underline{z}} = A\underline{z} + Bu \text{ and } y = C\underline{z} + Du \qquad (2)$$

where u(t) = control input, y(t) = output variable, A = State matrix, B = input matrix, C = Output matrix and D = input-to-output coupling matrix.

3. H Infinity Control Design Method

The key idea of H-infinity design is to synthesize optimal full information controller that minimizes infinity norm of the closed loop system between disturbance and input. The optimal control problem is considered as a dynamic optimization problem, a mini-max problem. It is under the condition that information of all states and disturbance input are available for feedback [5-6]. The objective function is defined as a cost function, J and optimization variables are disturbance, $w(t)$ and output, $y(t)$ as shown in Eq.(4). Also, the state space of the system is considered as dynamic constraints. This can be shown as follows.

$$J = \|G_{yw}\|_{\infty,[0,t_f]} = \sup_{\|w(t)\|_{2,[0,t_f]} \neq 0} \frac{\|y(t)\|_{2,[0,t_f]}}{\|w(t)\|_{2,[0,t_f]}} < \gamma \qquad (3)$$

$$\dot{\underline{z}} = A\underline{z} + B_u u + B_w w \text{ and } y = C_y \underline{z} + D_{yu} u \qquad (4)$$

After solving the Hamiltonian equation corresponding to from Eq.(3) and Eq.(4), the suboptimal control can be determined as

$$u(t) = K(t)\underline{z}(t) \qquad (5)$$

where $K(t) = B_u{}^T P$ and a matrix $P(t)$ is the solution of the Riccati differential equation as shown in Eq(6).

$$PA + A^T P - P(B_u B_u{}^T - \gamma B_w B_w{}^T)P + C_y{}^T C_y = 0 \qquad (6)$$

for H_∞ sub optimal control under the condition that

$$A - (B_u B_u{}^T - \gamma^2 B_w B_w{}^T)P \qquad (7)$$

is stable. The details are proposed in many textbooks Burl [5], Helton et al.[3], Tsachouridis et al.[4], and Skpgestal and Postlethwaite [6].

4. Simulation and Results

The feasibility of H-infinity controller design method can be shown by the simulation results of applying H-infinity controller to stabilize the example of a linear spring connected double inverted pendulum under the disturbance. Then, the simulation results of H infinity method and the LQR method are compared and discussed. The simulation is implemented in MATLAB software. The example of a linear spring connected double inverted pendulum system is defined by specifying the parameters of the system as follows:

$m_1 = 0.25$ kg, $m_2 = 0.25$ kg, $m_3 = 1.5$ kg, $m_4 = 1.5$ kg, $c_0 = 0$, $c = 0$, $l_1 = 0.4$ m, $J_{C1} = 0.0033$ kg.m², $l_{C1} = 0.2$ m, $c_1 = 0.05$, $l_2 = 0.4$ m, $J_{C2} = 0.0033$ kg.m², $l_{C2} = 0.2$ m, $c_2 = 0.05$, $k = 100$ N/m, $g = 9.81$ m/s² in appropriate SI units.

Both controller design methods are tested by two disturbance signals which are bound energy. First, disturbance is in the form of Eq.(8.1) and Eq.(8.2).

$$w_1(t)=\begin{cases}0, t\le a\\ \dfrac{t-a}{b-a}, a\le t\le b\\ \dfrac{c-t}{c-b}, b\le t\le c\\ 0, c\le t\end{cases} \tag{8.1}$$

where, $\overline{A}=1$, $a=3, b=6$, and $c=9$ as shown in Fig 2.

$$w_2(t)=e^{-at}\overline{A}\sin(\sqrt{b}t+\phi) \tag{8.2}$$

where, $\overline{A}=1$, $a=0.8$, $b=10$, and $\phi=0$ as shown in Fig 2.

In the first situation the system is affected by a disturbance as Eq.(8.1). The H-infinity feedback control system is stable, since all poles or eigenvalues of $A-(B_uB_u^T-\gamma^2B_wB_w^T)P$ are located on the left half plane as shown in Fig 3. The numerical solution of Riccati's Equation is shown in Eq.(9).

$$P=\begin{bmatrix}0.0001 & -0.0001 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000\\ -0.0001 & 0.0001 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000\\ -0.0000 & -0.0000 & 1.1032 & -0.5402 & -0.0000 & -0.0025 & 0.0779 & -0.0132\\ 0.0000 & 0.0000 & -0.5402 & 0.2693 & 0.0000 & 0.0010 & -0.0376 & 0.0072\\ -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000\\ -0.0000 & 0.0000 & -0.0025 & 0.0010 & -0.0000 & 0.0006 & -0.0002 & 0.0000\\ -0.0000 & -0.0000 & 0.0779 & -0.0376 & -0.0000 & -0.0002 & 0.0056 & -0.0008\\ 0.0000 & 0.0000 & -0.0132 & 0.0072 & 0.0000 & 0.0000 & -0.0008 & 0.0003\end{bmatrix}10^5 \tag{9}$$

Eigenvalues of P, $\lambda(P)$, are

$$\lambda(P)=\{1.9252e-004, 0.0060, 0.1342, 1.0737, 15.1359, 56.7692, 408.5991, 1.3742e+005\} \tag{10}$$

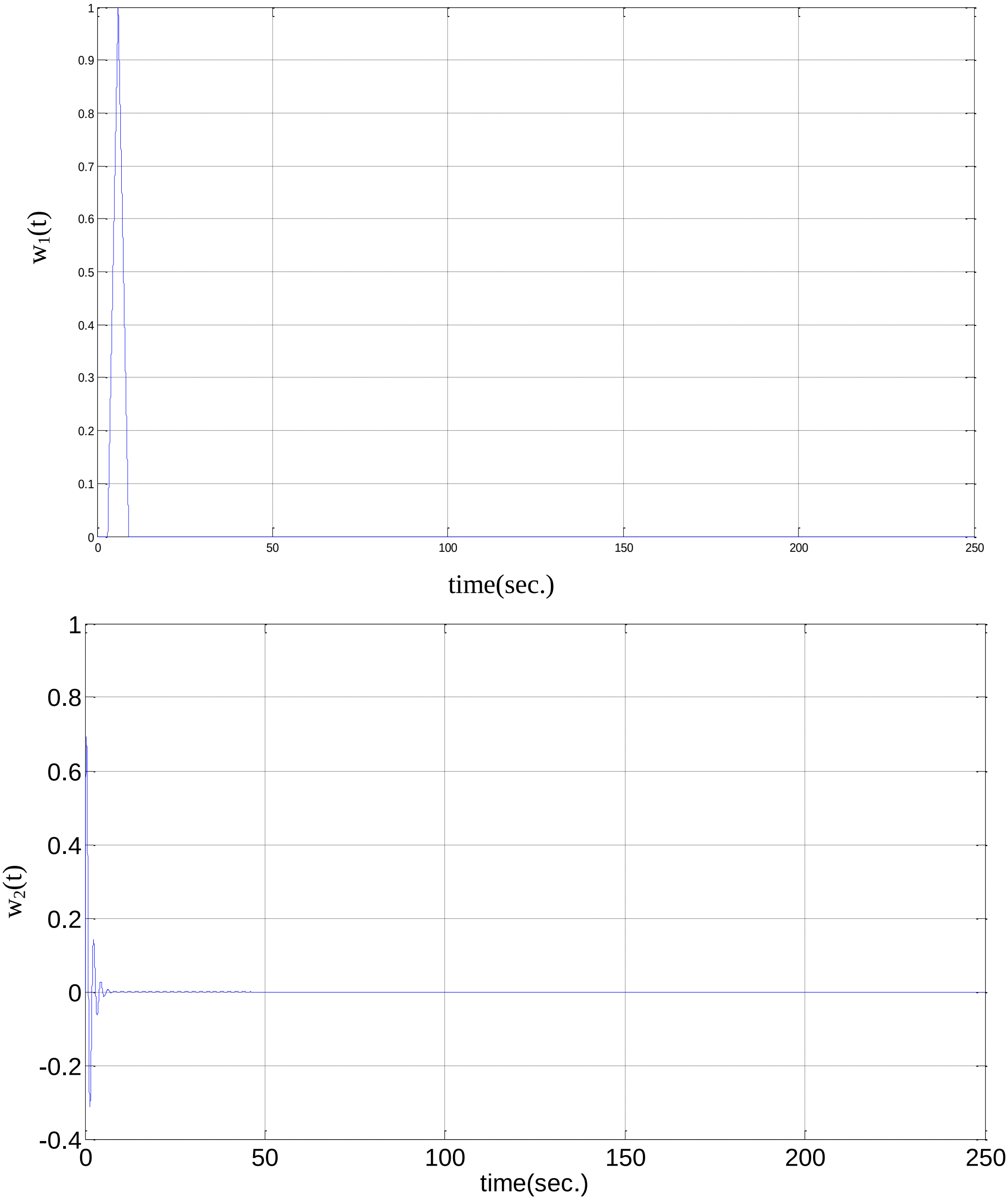


Fig. 2. (a) A Disturbance signal, $w_1(t)$; (b) A Disturbance signal, $w_2(t)$

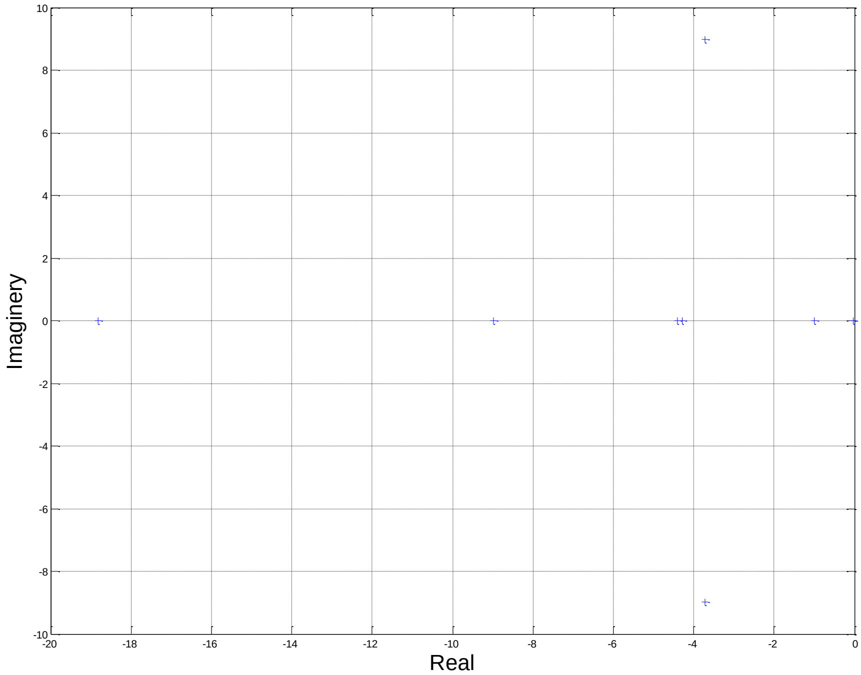


Fig. 3. Poles of the H-infinity feedback control system

The control input signal of H-infinity controller is shown in Fig 4. All time response of state variables of H-infinity and LQR methods under the disturbance signal are presented in Fig 3. The blue dash curves and magenta curves in Fig 3 represent the time responses of all state variables corresponding to H-infinity and LQR respectively. The time responses of state variables of Z_1, $Z_{2,}$, Z_3, Z_4, Z_5 and Z_6, corresponding of to H-infinity and LQR method approach to zero at steady state. However, the time responses of Z_7 and Z_8 given by both methods oscillate with bound around the equilibrium point at zero, and the amplitudes of oscillation of H-infinity method are smaller than those of LQR method slightly. Considering time responses of state variables Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7 and Z_8, H-infinity and LQR controller can stabilize the system since the steady state responses of each state variable from both methods are almost the same.

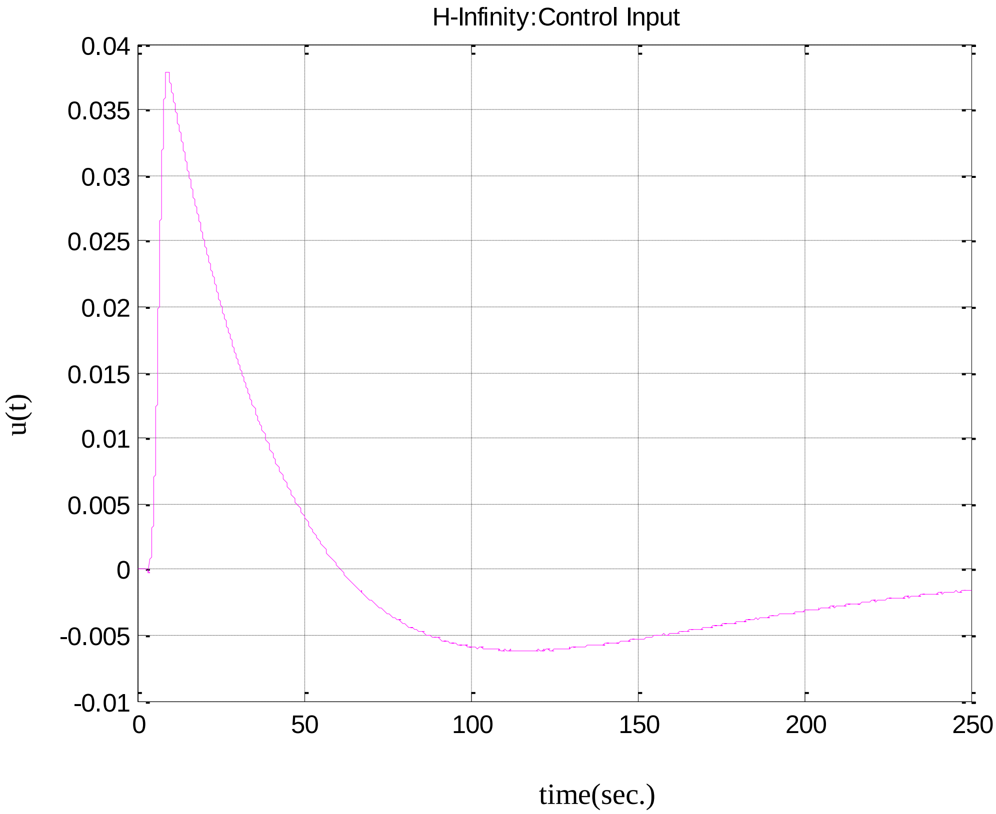
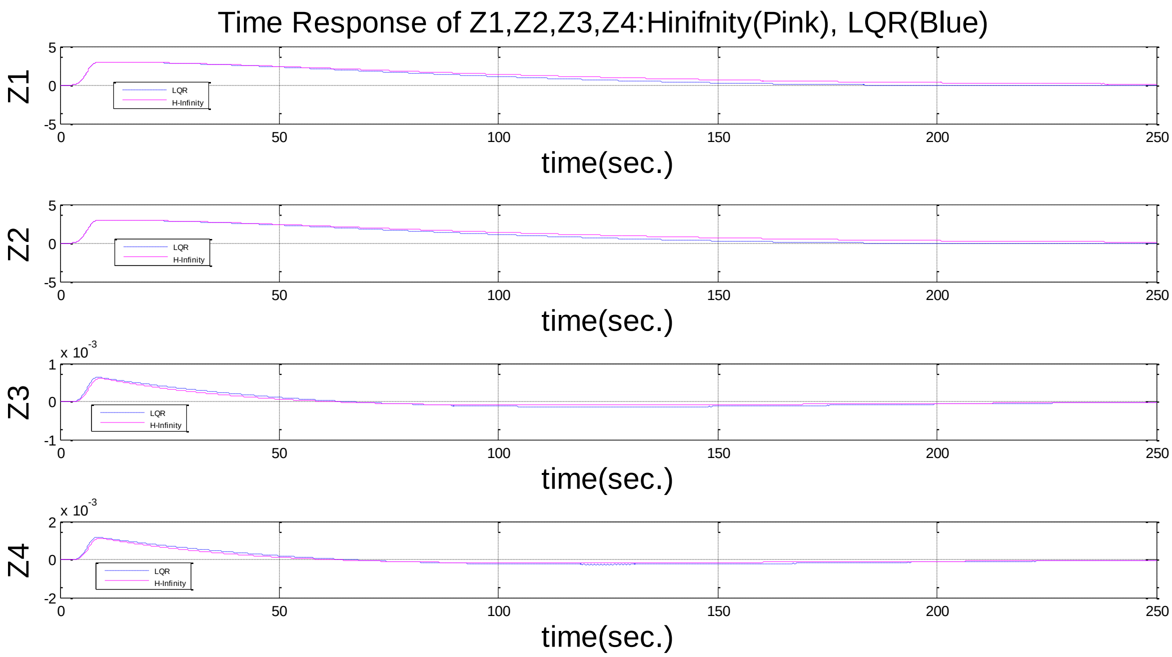


Fig. 4. control input signal of H-infinity feedback control system under a disturbance signal, $w_1(t)$



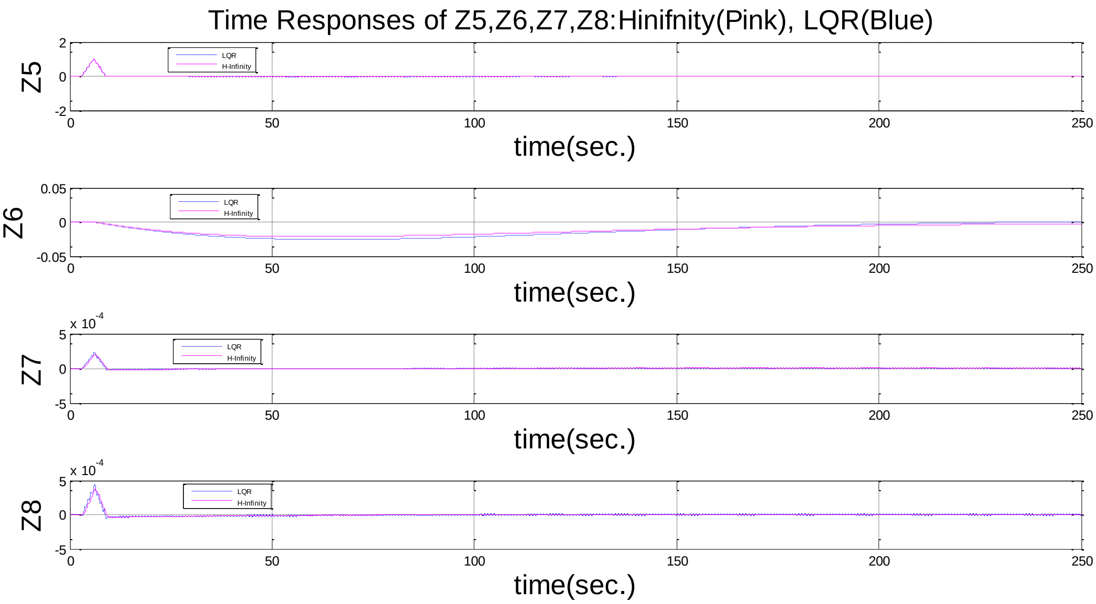


Fig. 5. (a) Time response of state variables of z_1, z_2, z_3 and z_4 of the H-infinity method (in blue dash curves) and the LQR method(in magenta curves) under a disturbance signal, w1(t);

(b) Time response of state variables of z_5, z_6, z_7 and z_8 of the H-infinity method (in blue dash curves) and the LQR method(in magenta curves) under a disturbance signal, w1(t)

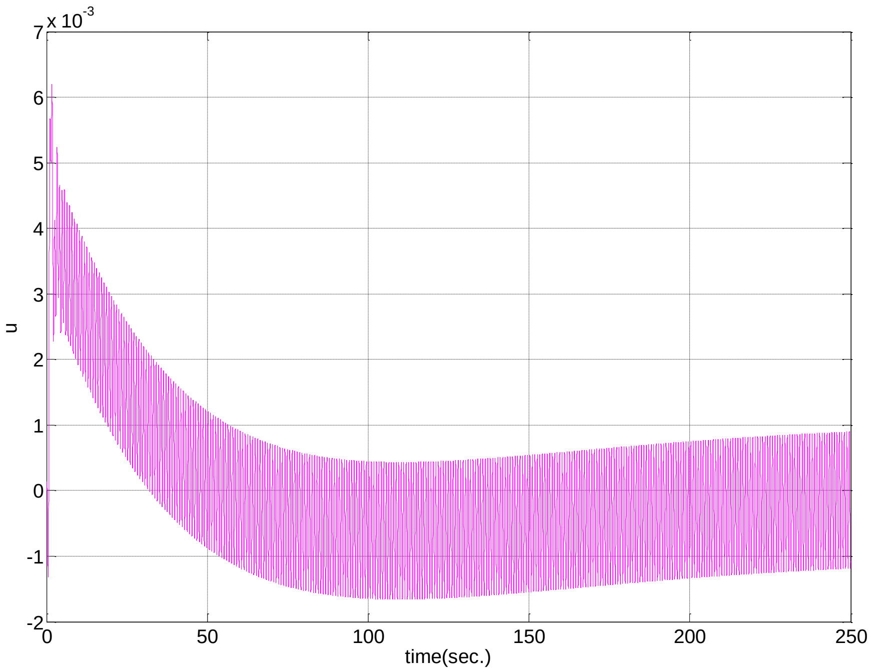


Fig. 6. A control input signal of H-infinity feedback control system under a disturbance signal, $w_2(t)$

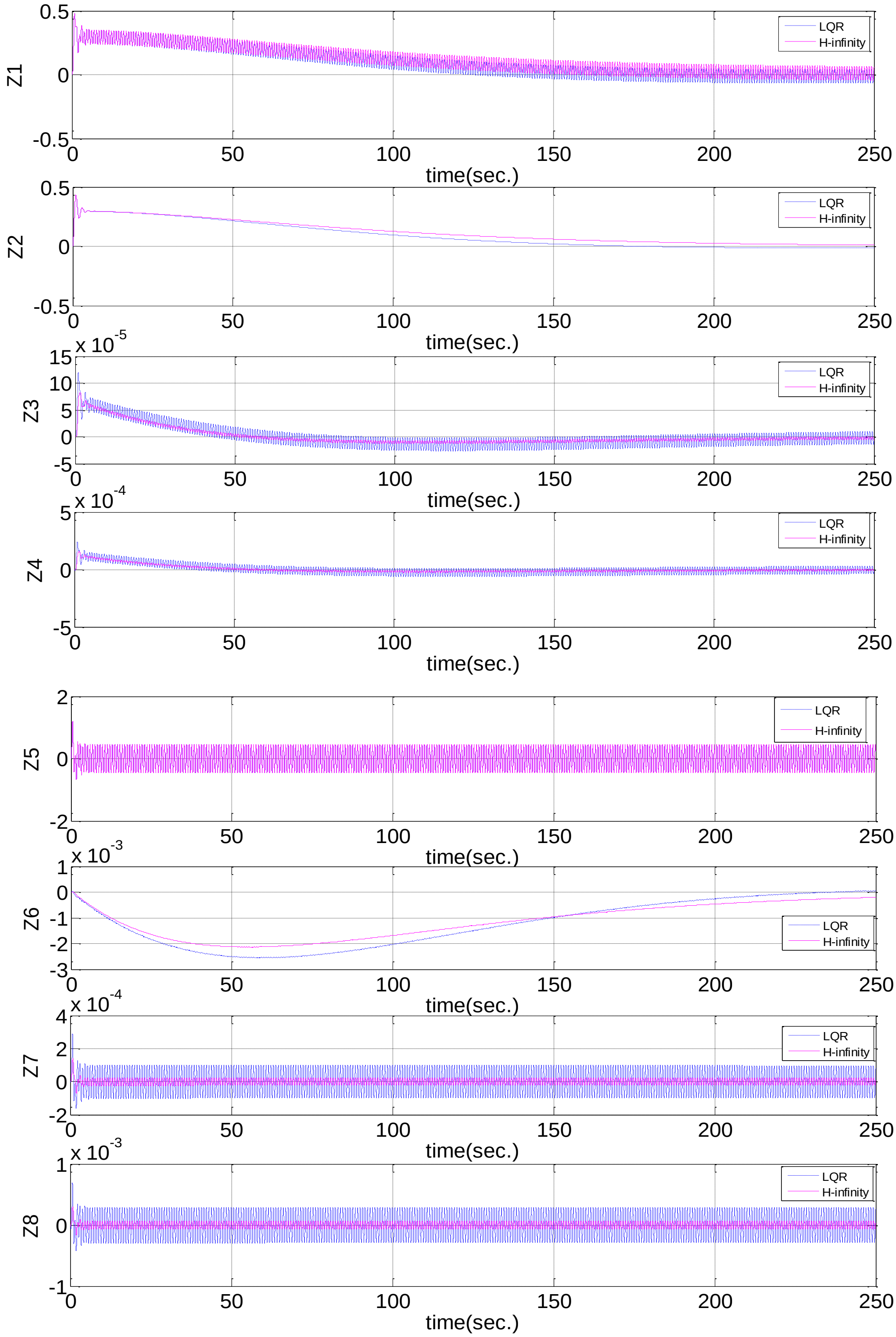

Z1
Z2
Z3
Z4
Z5
Z6
Z7
Z8
x 10^-5
x 10^-4
x 10^-3
x 10^-4
x 10^-3
time(sec.)
LQR
H-infinity

Fig. 7. (a) Time response of state variables of z_1, z_2, z_3 and z_4 of the H-infinity method (in blue dash curves) and the LQR method(in magenta curves) under a disturbance signal, w2(t);

(b) Time response of state variables of z_5, z_6, z_7 and z_8 of the H-infinity method (in blue dash curves) and the LQR method(in magenta curves) under a disturbance signal, w2(t)

5. Conclusions

H-infinity controller is feasible and appropriate to control and stabilize a mass spring connected double inverted pendulum system under the finite energy disturbance as shown and discussed above. Comparison between H-infinity controller and LQR controller shows that H-infinity controller performs better than LQR does.

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