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Root mean square calculations of periodic functions by sampling method

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Abstract

This paper proposes a technique of sampling method to calculate root mean square (RMS) of periodic functions with error analysis. This proposed method will be quite useful for computational technique which is done by running a sequential time domain with exact resolutions to generate discrete values of periodic functions. Error analysis from arbitrary data collecting of sinusoidal functions will be standardized to apply for any function. In the end, the errors from this method have been shown that they will be approached to the result computed from the calculus integration technique when the number of sampling has been increased. From the experiment, we can find the formula of error analysis to calculate correct RMS by using low sampling size.

Keywords: Root mean square, periodic function, sampling method

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1. Introduction

The root mean square mathematical operation is widely used in power engineering.[1], as an example, it has been mentioned to be equivalent to direct current signal. RMS has been used as the representative for an alternative current signal, which fluctuates with time domain. This paper proposes a technique of samplings methods to calculate root mean square (RMS) of a periodic function with error estimation formula, it will be better than a complicate calculus integration technique which has never mentioned about error estimation formula. The interesting point of the paper is the discovery of error estimation formula for sinusoidal function, which may be used as the basic of all periodic functions when the Fourier's series analysis is applied to distribute the periodic functions.

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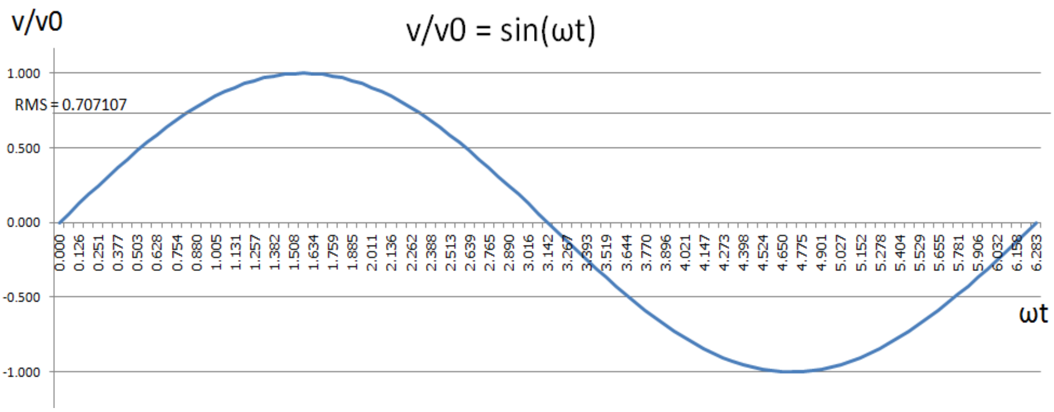

Fig. 1. The figure shows us that RMS equals 0.707107

Generally calculus integration technique is a familiar technique to calculate RMS value of any periodic function according to equation (1) as definition of RMS[1].

$$RMS = \sqrt{\frac{1}{T}\int_{T_1}^{T_1+T} f^2(t)dt} \tag{1}$$

or

$$RMS^2 = \frac{1}{T}\int_{T_1}^{T_1+T} f^2(t)dt \tag{2}$$

where RMS is the root mean square value of function *f*(t) and T is its period.

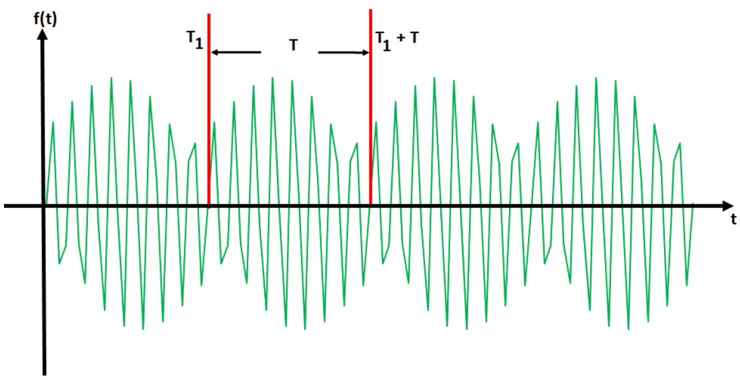

Fig. 2. A periodic function *f*(t) has a period of T. The root mean square value will be calculated between T_1 and $T_1 + T$

Refer to the definition of expected value or mean of function, it can be shown as following.

$$\left\langle f(t)\right\rangle = \frac{1}{T}\int_{T_1}^{T_1+T} f(t)dt \tag{3}$$

Therefore, the expected value of $f^2(t)$ is

$$\left\langle f^2(t)\right\rangle = \frac{1}{T}\int_{T_1}^{T_1+T} f^2(t)dt \,. \tag{4}$$

From relation (2), RMS^2 can be rewritten as

$$RMS^2 = \left\langle f^2(t)\right\rangle . \tag{5}$$

From relations of Eq. (3) and (5), the expected values of some sinusoidal functions can be calculated (with the same period of T) as shown in Table 1.

Table 1. The expected, RMS^2 and RMS values of some sinusoidal functions. (It have been founded that the RMS value always equals to $\sqrt{0.50}$ (≈ 0.707107) exactly.)

$f(t)$	$\left\langle f(t)\right\rangle$	$RMS^2 = \left\langle f^2(t)\right\rangle$	RMS	*Remark*
$\sin(at)$	0.000000	0.500000	0.707107	--
$\cos(bt)$	0.000000	0.500000	0.707107	--
$\sin(at)\cos(bt)$	0.000000	0.500000	0.707107	--
$\sin(at)\sin(bt)$	0.000000	0.500000	0.707107	$a \neq b$
$\cos(at)\cos(bt)$	0.000000	0.500000	0.707107	$a \neq b$

Sometime, calculated periodic functions are too complicate to use integration techniques such as sinusoidal functions multiplied by polynomial or exponential terms, or some functions, which will not be convenient to be integrated, include some function that cannot be integrated. Then the sampling method with easy root mean square technique may be used as another option to do.

2. Methodology

Normally, the purposed sampling method is a systematic sampling[3] to collect data from generating of periodic functions, then the easy mean square of all such generated data will be used to calculate RMS in the next step. The Fig. 2 shows the flowchart of the procedure to calculate RMS by using these methods.

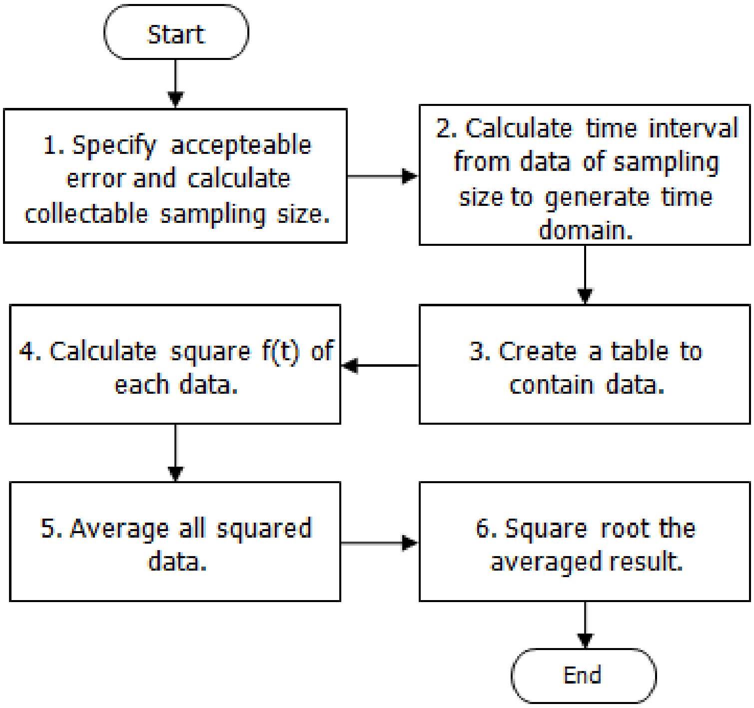

Fig. 3. The flowchart of the procedure of RMS calculation

In our methods, the experiment introduces some formulas to get acceptable errors, especially for sinusoidal functions, which are fundamental functions to comprise with any periodic function.

First, sinusoidal functions are primitive functions to comprise any periodic functions according to Fourier'series concept, *f*(t) = *sin*(t) was chosen to be the 1st function in our experiment to study RMS calculation.

Then, in the case of other periodic functions, which are not sinusoidal forms, the Fourier'series concept can be used to generate sinusoidal harmonic terms according to following formula and each term can be analyzed to calculate exact value of RMS[2].

$$f(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos \frac{2k\pi t}{T} + b_k \sin \frac{2k\pi t}{T}) \quad (6)$$

3. Results and Discussion

The experimental results of sampling size n = 4 and n = 10 are shown in Table 2 and 3, respectively. However, regarding to the experiments has been conducted in many times until we have found the relation of the error versus sampling size. Figure 4 shows the variation of sampling size only for n = 4. The results of error values for sampling size n=4 to n=50 are plotted as shown in Fig. 5.

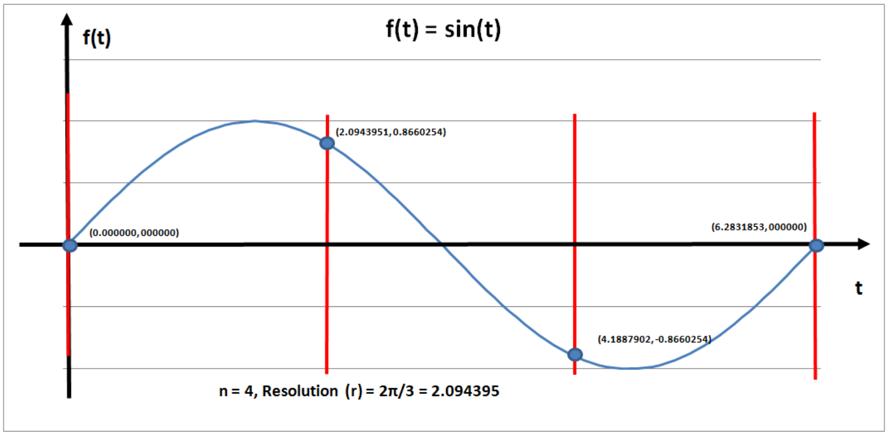

Fig. 4. Data were collected from f(t) = sin(t)
(Calculated RMS : $(\text{Average})^{0.5}$, PPM Error : (Standard RMS – Calculated RMS) x 1000000 / Standard RMS)

Table 2. The sampling size (n) = 4, period (T) = 6.2831853, resolution of time (r) = T/(n-1) = 1.0471976, Using 4 values of t which provide 4 values of f(t) to calculate RMS

n = 4 f(t) = sin(t) T = 6.2831853 r = T/(n-1)

t	tm = t1 + (m-1)r	sin(t)	$\sin^2(t)$
t1	0.000000	0.000000	0.000000
t2	2.094395	0.866025	0.750000
t3	4.188790	-0.866025	0.750000
t4	6.283185	0.000000	0.000000

Average	0.375000
Calculated RMS	0.612372
Standard RMS	0.707107
Error	0.094734
PPM Error	133975

Table 3. The sampling size (n) = 10, period (T) = 6.2831853, resolution of time (r) = T/(n-1) = 0.6983132, Using 10 values of t will provide 10 values of f(t) to calculate RMS

n = 10 f(t) = sin(t) T = 6.2831853 r = T/(n-1)

t	tm = t1 + (m-1)r	sin(t)	$\sin^2(t)$
t1	0.000000	0.000000	0.000000
t2	0.698132	0.642788	0.413176
t3	1.396263	0.984808	0.969846
t4	2.094395	0.866025	0.750000
t5	2.792527	0.342020	0.116978
t6	3.490659	-0.342020	0.116978
t7	4.188790	-0.866025	0.750000
t8	4.886922	-0.984808	0.969846
t9	5.585054	-0.642788	0.413176
t10	6.283185	0.000000	0.000000

Average	0.450000
Calculated RMS	0.670820
Standard RMS	0.707107
Error	0.036286
PPM Error	51317

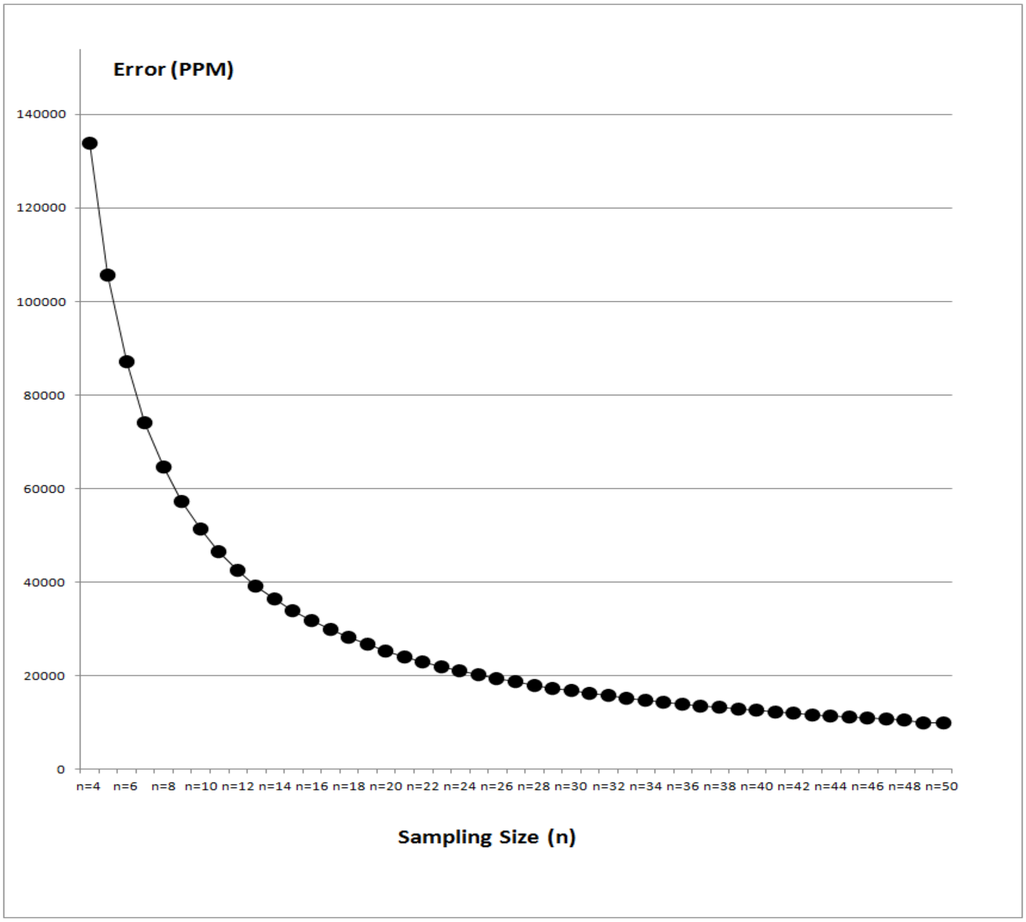

Fig. 5. The graph of error versus sampling size (n) of a sinusoidal function

4. Conclusion

From the number of experiments, this method may be a new option to calculate root mean square. In addition, the error from this method for sinusoidal function has been related to sampling size (*n*) according to the following equation.

$$Error = \sqrt{0.5} - \sqrt{0.5 - \frac{1}{2n}} \qquad (7)$$

After the error formula have been found, exact value of RMS could be given as

$$RMS = RMS_{stat} + Error \qquad (8)$$

RMS_{stat} is RMS from sampling method.

For further application of this error formula, we may use it for general periodic functions, which are summarized from orthogonal terms of Fourier'series.

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